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# **Making Sense of How Students Come to an Understanding of Physics: An Example from Mechanical Waves**



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## ABSTRACT

While physics education research (PER) has traditionally focused on introductory physics, little work has been done to organize and develop a model of how student come to make sense of the material they learn. By understanding how students build their knowledge of a specific topic, we can develop effective instructional materials. In this dissertation, I describe an investigation of student understanding of mechanical and sound waves, how we organize our findings, and how our results lead to the development of curriculum materials used in the classroom.

The physics of mechanical and sound waves at the introductory level (using the small-amplitude approximation in a dispersionless system) involves fundamental concepts that are difficult for many students. These include: distinguishing between medium properties and boundary conditions, recognizing local phenomena (e.g. superposition) in extended systems, using mathematical functions of two variables, and interpreting and applying the mathematics of waves in a variety of settings. Student understanding of these topics is described in the context of wave propagation, superposition, use of mathematics, and other topics. Investigations were carried out using the common tools of PER, including free response, multiple-choice, multiple-response, and semi-guided individual interview questions.

Student reasoning is described in terms of primitives generally used to simplify reasoning about complicated topics. I introduce a previously undocumented primitive, the object as point primitive. We organize student descriptions of wave physics around the idea of patterns of associations that use common primitive elements of reasoning. We can describe students as if they make an analogy toward Newtonian particle physics. The analogy guides students toward describing a wave as if it were a point particle described by certain unique parts of the wave. A diagnostic test has been developed to probe the dynamics of student reasoning during the course of instruction.

We have replaced traditional recitation instruction with curriculum materials designed to help students come to a more complete and appropriate understanding of wave physics. We find that the research-based instructional materials are more effective than the traditional lecture setting in helping students apply appropriate reasoning elements to the physics of waves.

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# Chapter 1: The Need for Systematic Investigation of Student Understanding of Physics

## Introduction

Investigations of student difficulties with physics are growing in number and sophistication. As researchers gain deeper insight into student understanding of the material taught in the classroom, they are able to create curriculum materials that are more effective in improving a student's actual understanding. At the same time, growing understanding of student reasoning provides the opportunity to find more systematic descriptions of how students come to make sense of the physics. By evaluating student performance in a modified curriculum setting, researchers can then develop an understanding of not only the curriculum's effects on student learning, but also the manner in which a curriculum *can* affect student learning.

There are many goals when investigating student understanding of physics. As researchers, we aim to recognize and “diagnose” specific difficulties while also finding the most common difficulties related to a specific topic. We try to help individual students more effectively while also creating curriculum that helps the highest number of students overcome the most common difficulties. The general goal is to help students understand what it means to understand physics.

In this dissertation, I will discuss the above ideas in the context of student difficulties with the physics of mechanical waves. The physics of mechanical waves is common to most introductory physics curricula at the university level and provides many interesting topics in which to discuss how students come to an understanding of physics. My investigations have taken place at the University of Maryland (UMd) with engineering students taking a required three-semester sequence of physics classes. The students discussed in this dissertation were in the second semester of the sequence.

In this chapter, I summarize the dissertation by giving an overview of the issues that affect the discussion of physics education research on student understanding of mechanical waves. Rather than discussing the contents of each chapter in order of its appearance, I will describe some of the issues that play a role in the dissertation while pointing out where a discussion of these issues can be found.

## Physics Education Research

The field of Physics Education Research (PER) has come about in reaction to the growing need for innovative methods in education that address student difficulties with the difficult material they are required to learn in our physics classrooms. PER involves

- investigations of specific aspects of student understanding of the physics,
- the development of investigative probes to help fulfill this goal,
- the development of statistical methods that help researchers organize, analyze, and present their findings,

- the design and implementation of curriculum materials that provide a more effective learning environment for students, and
- evaluations of the effectiveness of these materials.

In chapter 2, I give an overview of PER by summarizing work done by some of the leaders in the field. In the course of the analysis, I discuss different methods of analysis that have been used, including focusing both on specific aspects of understanding only and on broader descriptions of common student difficulties. In addition, I discuss how PER can lead to curriculum materials that can be demonstrated to be effective in addressing student difficulties with the physics. The remainder of the dissertation presents my own work, done as part of the Physics Education Research Group (PERG) at UMd, which involved all the aspects of PER as described above.

## Wave Physics

In order to discuss student understanding of wave physics, it is necessary to describe in detail what it is that we want them to actually learn in our classes. A mechanical wave is a propagating disturbance to a system, such as a wave traveling on a long, taut string. At the introductory level, we teach students a very simplified model of waves. In this model, there are no large amplitude waves and there is no dispersion in the system. Any disturbance will propagate indefinitely. Mathematically, these traveling waves can be described by functions of the form  $f(x \pm vt)$ , where  $f$  is any function that describes the displacement of the mechanical wave from equilibrium.

In chapter 2, I discuss the generally accepted model of waves that is taught at the introductory level. Many of the ideas of wave physics are subtle and rarely addressed explicitly in textbooks. For example, an understanding of waves requires an understanding of the role of initial conditions to help create a wave, though the initial conditions do not affect the manner of the wave's propagation through the system. Also, propagation occurs due to local interactions between "nearest neighbors" in the system. Student understanding of the distinctions between local phenomena and global phenomena plays a central role in this dissertation.

After the discussion of the physics, I discuss previous research by summarizing the literature of investigations into student difficulties with wave physics. Very few studies have been published, and the common themes among those that are available suggests that wave physics is a rich topic for investigating more general patterns of reasoning that we find in students.

## Student Difficulties with Wave Physics

The wave physics topics discussed in this dissertation include

- wave propagation speed (and how to change it),
- wave superposition (point-by-point addition of displacement),
- the mathematical description of waves (the  $f(x \pm vt)$  dependence), and
- the physics of propagating sound waves.

In addition, other topics play a role, such as wave reflection, though these have not been investigated in as great detail.

The specific wave physics topics serve as a context in which to discuss concepts and ideas that are more general to physics as a whole. These general ideas (displacement from equilibrium, for example, or the role of initial conditions in describing the dynamics of a system) build on concepts that students have encountered in their previous mechanics classes and also play a role in students' future studies. As a result, the discussion of student difficulties in chapter 3 provides a context in which to discuss how students build on the knowledge that they bring into the classroom.

## Organizing Student Difficulties

In addition to investigating student difficulties with specific physics concepts, we must also try to find ways to organize, explain, and discuss systematically how students are coming to their understanding of the material. An extensive literature has grown in the fields of education and cognitive studies in which these issues are addressed. In chapter 4, I give a summary of some of these ideas that help account for some of the difficulties we see students having with wave physics. Each of the different cognitive concepts that I discuss is presented with a typical example of how student reasoning has been interpreted in physics through the use of these different ideas. By presenting these cognitive ideas, I suggest a model of learning that is applicable to describing student difficulties with waves.

In chapter 5, I apply the proposed model of learning to the specific student difficulties first presented in chapter 3. We describe student reasoning in terms of fundamental and very simple ideas that are consistently and generally applied to many different situations. These *reasoning primitives* are generally applicable to many different situations in many different (possibly non-physics) settings. In physics, the same primitive may be applied in contradictory ways to a single physical situation. We refer to the application of a primitive in a context as a facet of reasoning.

In chapter 5, I discuss a primitive not previously presented in the literature, the *object as point* primitive. Students often apply this and other useful and reasonable primitives (helpful in mechanics, for example) inappropriately when thinking about the topic of wave physics. This provides us with a context in which to discuss different aspects of student reasoning. On the one hand, it is encouraging to see students trying to make sense of new topics in terms of the material that they have learned in previous semesters (though well-researched difficulties from mechanics arise again). On the other hand, we find that students do not have the ability to determine whether the primitives that they apply to a situation are appropriate in that setting.

Furthermore, many students seem to inappropriately use a set of primitives in conjunction with each other. We describe this as a pattern of association, meaning that students seem to use more than one primitive inappropriately to describe a single topic, but we do not claim that this is a robust model that students have in their head. The pattern of association may serve to help guide a student's choice of primitives in a given context. We refer to the pattern of association that many students use when discussing the physics of mechanical waves as the *Particle Pulses Pattern of*

*Association*, because student responses indicate a simplification of finite length waves to single points (rather than extended regions of displacement). In addition, students show great difficulties reasoning about force and motion with waves.

## **Designing Curriculum to Address Student Needs**

To help students in their learning of physics, we have begun to develop and implement a set of instructional materials that are designed to address specific student difficulties with the material. These materials, known as *tutorials*, are based on a design by the University of Washington (UW) Physics Education Group, under the guidance of Lillian McDermott. In tutorials, students participate in a process in which we

- *elicit* their difficulties, through questions that ask for their predictions or descriptions of a physical situation,
- *confront* students with a weak understanding of the material with evidence to show that their predictions were incorrect, and
- help students *resolve* their difficulties through guided questions and activities designed to let the student build their own robust understanding of the material.

The process of the development of research-based curriculum materials is described in chapter 2 in the context of a UW tutorial. Tutorials developed at UMD to address student difficulties with wave physics are described in chapter 6. This description includes a summary of how the computerized videos used in the instructional materials were created. In addition, in chapter 6 data are presented to indicate how effective the curriculum materials have been in addressing student difficulties. We have compared student performance from the beginning of an instructional sequence (pre-instruction on waves), the middle of the instructional sequence (after lecture instruction, but before tutorial instruction), and the end of instruction (after all instruction in the class has been completed). We find that student performance improves greatly as a result of research-based curriculum materials.

## **Investigating the Dynamics of Student Reasoning**

To better describe how student reasoning about the physics changes over the course of instruction, we have developed a diagnostic test that allows us to gain deeper insight into student reasoning on many different aspects of the physics at once. By using many questions that ask about the same topic, we are able to see the extent to which students reason consistently about the physics. By using the same questions before and after instruction, we are able to compare the development of student reasoning as a result of different instructional materials.

We find that students are not consistent in their reasoning. We can describe two patterns that students use to guide their reasoning: the Particle Pulses Pattern of Association, mentioned above, and the community consensus model of waves. Through the use of a short diagnostic test, we have been able to describe the dynamics of student reasoning as moving from a primarily incorrect application of otherwise useful and reasonable primitives to a state where they use both types of reasoning.

The implications for instruction in physics are that students leave our classes with an incomplete understanding of when to use which reasoning while thinking about the physics. I discuss the dynamics of student reasoning in chapter 7 of this dissertation.